Energy error based numerical algorithms for Cauchy problems for nonlinear elliptic or time dependent operators

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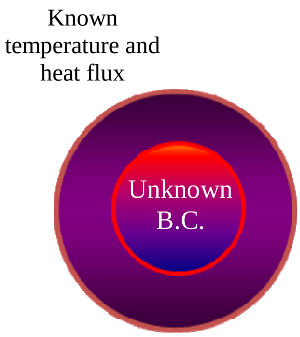
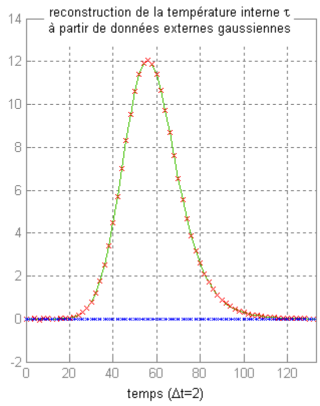
The Cauchy problem addressed here consists in finding a field solution of a PDE operator in a domain Ω, provided overspecified data are available on a part Γ_m of its boundary, whereas no information is available on the complementary part Γ_u of it. Applications of the Cauchy problems are numerous when the problem is seen as a boundary condition identification problem or as an extension problem inside a solid from data available on a part of its external surface. For various operators arising in transient and stationary heat transfer, linear and non linear elasticity or elastodynamics, a general solution method has been developed and implemented in 2D and 3D situations. The method relies on the introduction of two well posed problems, constructed with one of the overspecified data on Γ_m and one of the searched boundary conditions on Γ_u. Then because the Cauchy problem is solved if the two fields $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$, solutions of the preceding problems, are equal, the second step is to derive an "error" functional between the two fields J($\boldsymbol{u}_1$, $\boldsymbol{u}_2$). Instead using usual least-squares functionals, we propose to use energy based functionals for symmetric operator (stationary heat conduction, elasticity, electric conduction) or evolution operators with symmetric spatial part (heat equation, elastodynamics) and error in constitutive equation for hyperelastic materials.

The Cauchy problem is then recast into an optimization problem with the unknown boundary conditions as control variables and the error functional as objective function. Various numerical applications will be presented.



The Cauchy problem for the identification of BC inside a pipe with overspecified B.C at the external boundary. Transient heat conduction



Identified (uniform) time-dependant temperature inside the pipe

References

Andrieux S., Baranger T.N. and Ben Abda A., 2006, Solving Cauchy problem by minimizing an energy-like functional, *Inverse Problems*, **22**, 115-133.

Andrieux S., Baranger T.N., 2008, An energy error-based method for the resolution of the Cauchy problem in 3D linear elasticity, *Computer Methods in Applied Mechanics and Engineering*, **197**, 9-12, 902-920,

Baranger T.N., Andrieux S., 2008**,** An optimization approach for the Cauchy problem in linear elasticity, *J. Multidisciplinary optimization*, **35,** 141-152.